

Ch-1 Real Numbers

**Real Numbers!** The collection of all rational numbers and irrational numbers. Real number is either rational or irrational. For example  $1, \frac{5}{2}, \sqrt{2}, 15, -5, \dots$

**Theorem (1.1) (Euclid's division lemma)**

Given positive integers  $a$  and  $b$ , there exist unique integers  $q$  and  $r$  satisfying  $a = bq + r, 0 \leq r < b$ .

Euclid division algorithm is used to find the HCF of two given integers.

**Euclid's Division Algorithm!**

To obtain the HCF of two positive integers  $a$  and  $b$ , with  $a > b$ , steps are as follows.

① Apply Euclid's division lemma to  $a$  and  $b$  and find the whole numbers  $q$  and  $r$  such that  $a = bq + r, 0 \leq r < b$ .

② if  $r = 0$  then HCF of  $a$  and  $b$  is  $b$ . If  $r \neq 0$  then apply Euclid division lemma to  $b$  and  $r$ .

③ Continue the process till the remainder is 0. The divisor at this stage will be the required HCF.

Now we try to solve the Ex 1.1

Q1 (i) To find the HCF of 135 and 225

$225 > 135$ , Applying Euclid's division lemma to

$$a = 225 \text{ and } b = 135$$

$$225 = 135 \times 1 + 90$$

$$135 = 90 \times 1 + 45$$

$$90 = 45 \times 2 + 0$$

$$\text{Here } R = 0, b = 45$$

$$\therefore \text{HCF is } 45$$

$$\begin{array}{r} 135 \overline{) 225} \text{ (1)} \\ \underline{-135} \\ 90 \\ 90 \overline{) 135} \text{ (1)} \\ \underline{-90} \\ 45 \\ 45 \overline{) 90} \text{ (2)} \\ \underline{-45} \\ 45 \\ 45 \overline{) 45} \text{ (1)} \\ \underline{-45} \\ 0 \text{ - R} \end{array}$$

Q.2 Ex.1

Let  $a$  be any positive odd integer and  $b = 6$  (2)

then by Euclid division lemma

$$a = 6q + r \quad \therefore r = 0, 1, 2, 3, 4, 5 \text{ as } (0 \leq r < b)$$

when  $r = 0 \Rightarrow a = 6q + 0 = 6q$  (i)

$$r = 1 \Rightarrow a = 6q + 1 \text{ (ii)}$$

$$r = 2 \Rightarrow a = 6q + 2 = 2(3q + 1) \text{ (iii)}$$

$$r = 3 \Rightarrow a = 6q + 3 \text{ (iv)}$$

$$r = 4 \Rightarrow a = 6q + 4 = 2(3q + 2) \text{ (v)}$$

$$r = 5 \Rightarrow a = 6q + 5 \text{ (vi)}$$

values of  $a$  in (i) (iii) and (iv) is even.

$\therefore$  any positive odd integer is of the form  $6q + 1$

$6q + 3$  and  $6q + 5$ .

Now try yourself: Ex.1 Q.1 (iii) and (iv), Q.3, 4, 5.

**Assignment:-** (1) Use Euclid division lemma find HCF of

(i) 240 and 6552 (ii) 1288 and 575 (iii) 615 and 154.

(iv) 1385 and 155 (v) 2750 and 325

Q.2 show that any positive even integer is of the form  $4q$ ,  $4q + 2$  or  $4q + 4$ , where  $q$  is some integer.

Q.3 if  $d$  is the HCF of 45 and 27 find 2 and 3 ratios by using  $d = 227x + 45y$ .

Q.4 show that any positive odd integer is of the form  $8q + 1$  or  $8q + 3$  or  $8q + 5$  or  $8q + 7$ , where  $q$  is some integer.

# Fundamental theorem of arithmetic!

(3)

Every composite number can be factorised (expressed) as a product of primes and this factorisation is unique apart from the order in which prime factors occur.

For example,  $48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$

$140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$

Prime factorisation of above composite numbers are unique.

$$\begin{array}{r|l} 2 & 48 \\ \hline & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 2 & 3 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

We can find HCF and LCM with the help of fundamental theorem of arithmetic any two or more positive integers.

$$\begin{array}{r|l} 2 & 140 \\ \hline & 70 \\ \hline 2 & 35 \\ \hline 5 & 7 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

## Algorithm

(i) Factorise the given positive integers in prime factors and express them as a product of powers of ascending order.

(ii) For HCF identify common prime factors and find smallest exponent of these common factors and multiply them.

(iii) For LCM find the greatest exponent of all prime factors and multiply them.

For any two positive integers a and b.

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

$$\Rightarrow \text{HCF}(a, b) = \frac{a \times b}{\text{LCM}(a, b)}, \quad \text{LCM}(a, b) = \frac{a \times b}{\text{HCF}(a, b)}$$

P.T.O.

Ex 1.2 Q1 (ii)  $3825 = 3 \times 3 \times 5 \times 5 \times 17$

or  
 $= 3^2 \times 5^2 \times 17$  Ans

(4)

$$\begin{array}{r} 5 \overline{) 3825} \\ \underline{5 \phantom{00} 765} \\ 3 \phantom{00} 153 \\ \underline{3 \phantom{00} 51} \\ 17 \phantom{00} 17 \\ \underline{17 \phantom{00} 0} \\ 1 \end{array}$$

Q.2 (ii)  $510 = 2 \times 3 \times 5 \times 17$

$92 = 2 \times 2 \times 23$   
 $= 2^2 \times 23$

$$\begin{array}{r} 2 \overline{) 510} \\ \underline{3 \phantom{00} 255} \\ 5 \phantom{00} 85 \\ \underline{17 \phantom{00} 17} \\ 1 \end{array}$$

$$\begin{array}{r} 2 \overline{) 92} \\ \underline{2 \phantom{00} 46} \\ 23 \phantom{00} 23 \\ \underline{23 \phantom{00} 0} \\ 1 \end{array}$$

HCF = 2

LCM =  $2^2 \times 3 \times 5 \times 17 \times 23$   
 $= 23460$

LCM  $\times$  HCF =  $510 \times 92$

$2 \times 23460 = 510 \times 92$

$46920 = 46920$  H.P

Q.4 HCF (306, 657) = 9

$\therefore$  LCM =  $\frac{a \times b}{\text{HCF}(a, b)}$

$\Rightarrow$  LCM (306, 657) =  $\frac{306 \times 657}{9} = 34 \times 657 = 22338$  Ans

Q.5  $6^m = (3 \times 2)^m = 3^m \times 2^m$

$6^m$  ends with digit 0 if it is divisible by 2 and 5 both. It is divisible by 2 but it is not divisible by 5. Since it has no prime factor 5,  $\therefore 6^m$  can not end with 0.  
P.T.O

## Homework.

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Ex 1.2 Q.1. (i) (iii) (iv) (v).

Q.2 (i) (iii)

Q.3, Q.6, Q.7.

## Assignment:

Q.1 Express each of the following as the product of its prime factors (i) 234 (ii) 945 (iii) 1771 (iv) 20570

Q.2 Find HCF and LCM of following numbers.

(i) 120, 175 (ii) 350, 875

(iii) 12, 15, 21 (iv) 85, 750, 350

Q.3 Show that  $12^n$  can not end with digit 0 for any natural number  $n$ .

Q.4 Why  $17 + 11 \times 13 \times 17 \times 19$  is a composite number; explain.